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On the calibration of Lévy driven time series with coupling distances – an application in paleoclimate

Abstract

This work aims at quantifying the maximal distance between the empirical law of a given time series and the law of the paths of model, which is given in terms of a scalar SDE driven by additive Lévy noise. Several authors [Hein, Imkeller, Pavlyukevich, 2009] and [Gairing, Imkeller, 2014] in the Mathematics and in the Physics literature [Ditlevsen, 1999] suggested that the time series under consideration contains an α-stable jump component for some $\alpha \approx 1.75$.

For this purpose we introduce a metric on the formal space of Lévy triplets, which is essentially based on a weighted Wasserstein distance of suitably renormalized tails of a Lévy measure. Due to its optimal coupling property, we call it *coupling distance*. In the main theorem of our article we prove an estimate between the laws in path space of two Lévy diffusions driven by two Lévy processes in terms of the coefficients of the equations and the coupling distance of their Lévy triplets.

Combining this theoretical result, with the efficient statistical estimation of the Wasserstein 2-distance we obtain a non-trivial goodness-of-fit estimate between the law of a given time-series in path space and SDEs driven by heavy-tailed Lévy noise in terms of the *coupling distance* of their Lévy measures.

These results show that the law of certain paleoclimate data is closes to the law of an asymmetric Lévy diffusion with polynomial tail with a polynomial decay of the Lévy measure. In addition we find small values for a tail parameters $\alpha \gg 2$, which hints to strongly non-stable Lévy jump diffusions.