STABILITY AND SPECTRAL ATTRIBUTES OF NUMERICAL SOLUTIONS OF RICHARDS' EQUATION

Álvaro A. Aldama and Ismael Herrera
Institute of Geophysics,
National Autonomous University of Mexico
Javier Aparicio
Independent Consultant

Session: Fluids, friction and rheology, in rocks and porous media
Preference: Oral Presentation

Richards' equation governs the Darcian motion of liquids in unsaturated porous media. Numerical solutions of the same equation exhibit unexpected behavior due to its nonlinear character. In this paper, the commonly employed time integration schemes based on the θ–approximation, combined with finite difference or finite element spatial discretizations are considered. Since Richards' equation is parabolic, classical (frozen coefficients) analyses predict that such schemes are unconditionally stable for $\theta \geq ½$. Nevertheless, there is computational evidence that instabilities occur for $\theta = ½$ (Crank-Nicolson scheme), and larger values of θ. To explain such unexpected behavior, an analysis properly accounting for the dominant effects of nonlinearities in the stability properties of the discrete Richards system, is performed. Such analysis is based on the Taylor-Fréchet expansion of the discrete operator, and a multiple scale and asymptotic approach. Stability conditions are thus determined, clearly showing that the Crank Nicolson scheme may be unconditionally *unstable* and, therefore, its use should be avoided. In addition, explicit stability conditions for the fully implicit (Euler backward) scheme are obtained. Spectral amplitude ratios and phase errors are also determined for this last scheme, as a means to assess its accuracy. Results of numerical experiments that confirm the theoretical findings are included.