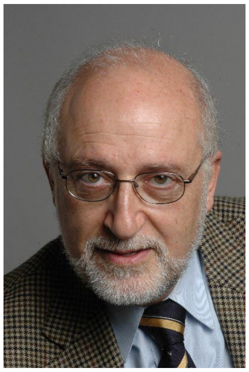

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Structural reliability, risk analysis, stochastic dynamics, earthquake engineering.

I first came to know of Luis Esteva in 1974, while a doctoral student at the University of Illinois at Urbana-Champaign. My interest in probabilistic methods had led me to the seminal paper by Rosenblueth and Esteva in 1972, which was the first to propose a code format based on the lognormal distribution – a format that is now omnipresent in many established or proposed probabilistic design procedures. At the time, Luis Esteva was a towering figure in my imagination, distant and unapproachable. It was not until 1980 that I met him personally at the 7^{th} World Conference in Earthquake Engineering in Istanbul, Turkey. The meeting was an eye opener: not only Professor Esteva was not a "towering" figure, he was undeniably friendly, approachable and kind. I do not remember when it was that I dared to drop the "Professor" title when addressing him, but his gentle and friendly attitude had a lot to do with it. Over the years I have enjoyed the pleasure of his company on many occasions, and have learned a lot from his papers and comments. The magnitude of his research accomplishments and service to the profession, the depth of his knowledge in probabilistic methods and earthquake engineering, and the breadth of his general knowledge are truly amazing. But what is even more significant is his humility, his ability to encourage young researchers, and, above all, his gentleness. In my imagination now, Luis Esteva is the quintessential Mexican gentleman and scholar.

Armen Der Kiureghian
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PROBABILITY CONCEPTS FOR PERFORMANCE-BASED EARTHQUAKE ENGINEERING

ABSTRACT

A framework formula for performance-based earthquake engineering has been proposed by the Pacific Earthquake Engineering Research Center (PEER). This paper addresses two aspects of that formula. One deals with the probabilistic information that one can obtain from the result of this formula, which is expressed in terms of the mean annual frequency of an earthquake effect exceeding a specified threshold. The other critically examines the predominant method for computing the probability distribution of a demand parameter for a given intensity of ground motion and suggests an alternative.

Introduction

Earthquake engineering, perhaps more than any other field of engineering, must confront and deal with uncertainties. The randomness in the occurrence of earthquakes in time and space, the vast uncertainty in predicting the intensities and other characteristics of the resulting ground motions, and large imperfections in the predictive models used to assess structural demands and capacities under cyclic loads, all compel us to make use of probabilistic methods in order to consistently account for the underlying uncertainties and make quantitative assessments of safety and reliability. Such analysis is also required for informed decision making for design, retrofit or maintenance of structures.

Luis Esteva has prominently contributed to the development and use of probabilistic methods to advance earthquake engineering. Indeed, he has contributed to many aspects of the field, including the development of probabilistic code formats (Rosenblueth and Esteva 1972), modeling and estimation of seismicity (Hasofer and Esteva 1985), seismic reliability assessment of structures (Esteva and Ruiz 1989, Esteva *et al*. 2001), development of optimal solutions for seismic instrumentation (Heredia-Zavoni and Esteva 1998, Heredia-Zavoni *et al*. 1999), performance-based seismic design criteria (Esteva *et al*. 2002) and maintenance (Montes-Iturrizaga *et al*. 2003), and selection of ground motion intensity measures for performance-based earthquake engineering (Giovenale *et al*. 2004). He has developed methods motivated by real-world needs, which are scientifically sound, yet simple and practical. His work has influenced the practice of earthquake engineering not only in Mexico, but also throughout the world. This is evidenced by the numerous citations to his papers, which one finds by searching the international earthquake engineering literature. One hopes and anticipates that his contribution and influence will continue for many years to come.

After many years of early development, the earthquake engineering community is finally moving towards incorporating probabilistic methods in a systematic way in the design and decision-making for earthquake effects on structures and other constructed facilities. One important manifestation of this is the effort in the Pacific Earthquake Engineering Research Center (PEER) for developing a performance-based earthquake engineering (PBEE) methodology. According to Moehle and Deierlein (2004), "performance-based earthquake engineering seeks to improve seismic risk decision-making through assessment and design methods that have a strong scientific basis and that express options in terms that enable stakeholders to make informed decisions. A key feature is the definition of performance metrics that are relevant to decision making for seismic risk mitigation." Implicit in this definition is the understanding that all relevant uncertainties in the design or decision-making are properly and consistently accounted for in computing the performance metrics.

As a contribution to this symposium honoring Luis Esteva, I have decided to discuss some probabilistic concepts related to the PEER PBEE methodology. Judging from Esteva's recent work, I believe this topic is of interest to him. After a brief introduction of the framework formula on which the methodology is based, I discuss the various types of probabilistic information that one can obtain from the mean annual frequency of occurrence, which is the main output of the formula. I then offer a critical discussion of the predominant method for computing the distribution of structural demand measures for a given intensity, which uses recorded ground motions. Potential shortcomings of the method are described and an alternative approach is suggested. It is hoped that these discussions enhance and refine the usefulness of the PEER PBEE methodology and framework formula.

The PEER PBEE Methodology

The PEER PBEE methodology is based on a framework formula that estimates the mean annual frequency of events where a specified variable exceeds a given threshold. Originally proposed by Cornell and Krawinkler (2000)[1], the formula has the form

$$\lambda(dv) = \int_{dm}\int_{edp}\int_{im} G(dv|dm)\left|\mathrm{d}G(dm|edp)\right|\left|\mathrm{d}G(edp|im)\right|\left|\mathrm{d}\lambda(im)\right| \qquad (1)$$

in which *im* denotes an intensity measure (e.g., the peak ground acceleration or the spectral acceleration at a selected frequency), *edp* denotes an engineering demand parameter (e.g., an interstory drift), *dm* denotes a damage measure (e.g., the accumulated plastic rotation at a joint), *dv* denotes a decision variable (e.g., dollar loss, duration of downtime), $G(x\,|\,y) = \Pr(x < X\,|\,Y = y)$ denotes the conditional complementary cumulative distribution function of random variable X given $Y = y$, $dG(x\,|\,y)$ is the differential of $G(x\,|\,y)$ with respect to x, and $\lambda(x)$ denotes the mean frequency of $\{x < X\}$ events per year. Absolute values are used on the three differential quantities since they are negative. The resulting quantity, $\lambda(dv)$, denoting the annual frequency of events where the decision variable *DV* exceeds the threshold *dv*, is the principal decision metric advocated by PEER for seismic risk mitigation. Similar formulas can be written for the

[1] The original version by Cornell and Krawinkler did not include the intermediate variable *EDP*. This variable and the corresponding integral were added in later publications of PEER.

annual frequencies $\lambda(dm)$ and $\lambda(edp)$ involving 2 and 1-dimensional integrals, respectively.

An important advantage of the framework formula (1) is that it decomposes the task of assessing the decision metric into the subtasks of seismic hazard analysis, $\lambda(im)$, structural response analysis, $G(edp|im)$, damage analysis, $G(dm|edp)$, and loss analysis $G(dv|dm)$, each of which may be handled by a different group of experts (Porter 2003, Moehle and Deierlein 2004). This decomposition is made possible through the fundamental assumption that, conditioned on *EDP*, *DM* is independent of *IM*, and, conditioned on *DM*, *DV* is independent of *EDP* and *IM*. Another fundamental assumption is that the structure is restored to its initial condition after each damaging earthquake event.

A number of investigators have suggested the use of formulas similar to (1) to compute the probability distribution of extreme *EDP*, *DM* or *DV* values for all earthquakes occurring during a specified period of time, typically one year or the lifetime of the structure. It has been shown in Der Kiureghian (2005) that, if there are non-ergodic uncertainties (i.e., uncertainties which do not renew at each earthquake occurrence, such as epistemic model uncertainties), then such use of the formula entails an error, which can be significant if the probability of interest is greater than around 0.01. Therefore, caution must be exercised in the use of (1) for computing probabilities. More on this is described below.

In the following two sections, two aspects of the PEER framework formula are discussed. The first deals with the types of information that one can gain from the mean annual frequency $\lambda(x)$, where x may denote a *DV*, *DM* or *EDP* threshold. The second deals with the methods available for computing the conditional distribution of an *EDP* for a given *IM*, i.e., the complementary CDF $G(edp \mid im)$.

What Can We Learn from $\lambda(x)$?

As mentioned earlier, $\lambda(x)$ denotes the mean annual frequency of occurrences of an earthquake effect X (e.g., a *DV*, *DM* or *EDP*) exceeding the threshold x. Assuming X is non-negative, a plot of $\lambda(x)$ versus x may appear as in Figure 1. Three distinct characteristics of this curve are noted:

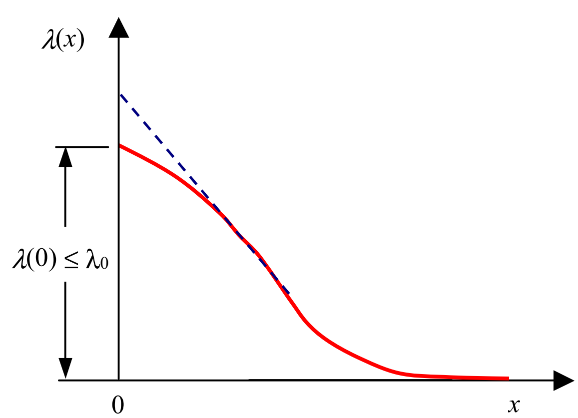

Figure 1. Typical plot of the annual frequency of occurrence of events $\{x < X\}$.

1. For X denoting a *DV*, *DM* or *EDP* quantity, $\lambda(x)$ is a non-increasing function of x.

2. As x approaches zero, $\lambda(x)$ approaches a finite value $\lambda(0)$. When X denotes an *EDP* quantity, $\lambda(0)$ is the mean annual frequency of all earthquakes considered in the hazard analysis of the site. We denote this value as λ_0. When X denotes a *DV* or *DM* quantity, $\lambda(0)$ is the annual frequency of earthquakes that cause finite damage and loss. Note that in this case $\lambda(0)$ can be smaller than λ_0, since some low-intensity earthquakes may not cause any damage or loss. In practice, one often computes $\lambda(x)$ for a few selected thresholds and uses extrapolation to construct the curve for small thresholds. For example, the tangent approximation shown in Figure 1 as a dashed line (usually in a semi-logarithmic plot) is often used. Obviously, the above interpretations of $\lambda(0)$ can be helpful in constructing a better approximation of $\lambda(x)$ for small thresholds.

3. The far tail of the curve normally drops off to zero for high thresholds, though for *DV* and *DM* quantities it is possible to imagine a sharp drop at a threshold $x_{\max}$ corresponding to the total damage or loss that can be sustained by the structure.

In addition to providing the mean annual frequency of events, the function $\lambda(x)$ provides the information described below.

Observe that $\lambda(x)$ denotes the mean number of earthquakes in a year that cause $\{x < X\}$, whereas λ_0 represents the mean number of all earthquakes in the same time period. It follows that the ratio $\lambda(x)/\lambda_0$ denotes the long-term fraction of earthquakes that produce an X exceeding the given threshold x. Thus, the function

$$F(x) = 1 - \frac{\lambda(x)}{\lambda_0} \tag{2}$$

represents the CDF of X for a randomly selected earthquake, and its derivative

$$f(x) = -\frac{1}{\lambda}\frac{d\lambda(x)}{dx} \tag{3}$$

represents the corresponding probability density function (PDF). Since $\lambda(0) \le \lambda_0$ (the inequality applying when X represents a *DV* or *DM* quantity), a plot of $F(x)$ may appear as in Figure 2. The discontinuity at $x = 0$ occurs when $\lambda(0) < \lambda_0$. As a result, the PDF of X, which is also depicted in Figure 2, includes a probability mass at $x = 0$, which is equal to the fraction of earthquakes that cause no damage or loss.

The probability distribution shown in Figure 2 is that of X for a randomly selected earthquake. Since a randomly selected earthquake is a lot more likely to have a low than a high intensity, this distribution is not the appropriate one for decision-making or for safety assessment. A quantity of interest for this purpose is the probability distribution of the largest X that can occur in a given period of time, T, say the life of the structure. We denote this by X_T. This distribution is of special interest when X denotes an *EDP* or *DM* quantity.

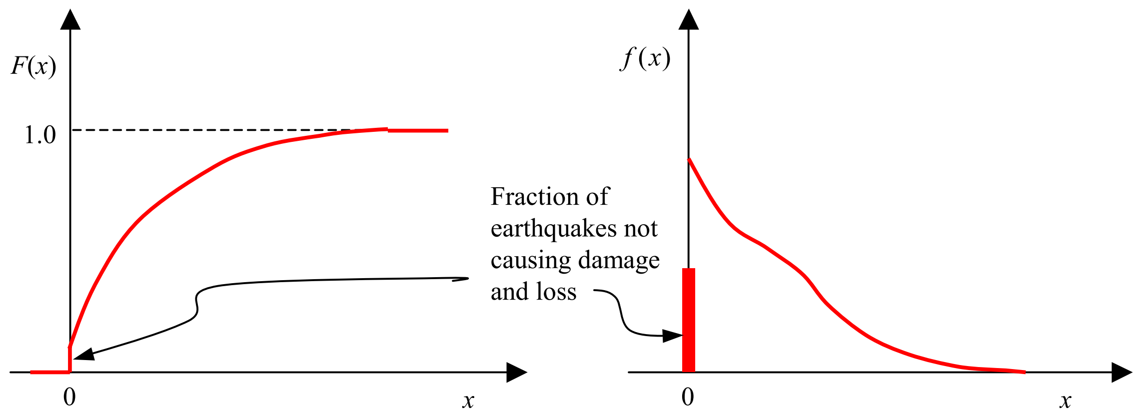

Figure 2. Conceptual plots of the CDF and PDF of X.

If the occurrences of $\{x < X\}$ events in successive earthquakes can be considered to be statistically independent, then an approximation of the CDF of X_T is given by

$$\Pr(X_T \leq x) \cong [F(x)]^{\lambda_0 T} \tag{4}$$

where $\lambda_0 T$ represents the mean number of events during T. The approximation lies in the fact that the mean number of events is used. Alternatively, again under the assumption of statistical independence, the occurrences of $\{x < X\}$ may be assumed to constitute Poisson events in time. In that case, the distribution of X_T is given by

$$\Pr(X_T \leq x) = \exp\{-\lambda_0 T[1 - F(x)]\} \tag{5}$$

In general, the distributions in (4) and (5) are nearly the same in the tail region of x.

Dependence between successive occurrences of events $\{x < X\}$, however, may occur when uncertainties are present, which do not renew at each earthquake event. These uncertainties, denoted "non-ergodic" in Der Kiureghian (2005), may arise from the unknown characteristics of the structure, or from epistemic uncertainties present in the assessment of seismic hazard or in the modeling of the structural response. A correct formulation of the distribution of X_T for that case is presented in Der Kiureghian (2005) and will not be described here. It is noted, however, that, in the presence of non-ergodic uncertainties, the approximations in (4) and (5) generally produce conservative results, i.e., underestimate the probability of the event $\{X_T \leq x\}$. This is because the statistical dependence between the successive $\{x < X\}$ events is usually characterized by a positive correlation. (If the capacity of the structure is on the low side, then it is so for all earthquakes.) One quick way to account for this effect is to replace $\lambda_0 T$ in (4) and (5) with a reduced value representing the mean number of *equivalent statistically independent events*. This kind of an approximation was used in Der Kiureghian (1980) in developing approximate expressions for the mean and standard deviation of the extreme of a random process. Numerical studies can be performed to determine the reduction factor for various levels of non-ergodic uncertainties, possibly as a function of x.

Now observe that for a variable X, the differential quantity $\lambda(x)-\lambda(x+\mathrm{d}x)=-\mathrm{d}\lambda(x)$ describes the mean number of events $\{x < X \leq x+\mathrm{d}x\}$ per year. Thus, the product $-x\,\mathrm{d}\lambda(x)$ describes the expected cumulative value of the outcomes of X in this differential range in one year. Integrating over the entire range, one obtains the expected cumulative value of all X values in one year

$$\begin{aligned} \mathrm{E}[\Sigma X] &= -\int_0^\infty x\,\mathrm{d}\lambda(x) \\ &= -x\lambda(x)\Big|_0^\infty + \int_0^\infty \lambda(x)\,\mathrm{d}x \\ &= \int_0^\infty \lambda(x)\,\mathrm{d}x \end{aligned} \qquad (6)$$

where, in the second line, we have used integration by parts. Thus, the area underneath the $\lambda(x)$ versus x curve gives the mean cumulative value of the X values for all earthquakes occurring in one year. Obviously, the quantity $\mathrm{E}[\Sigma X]$ is of great value when X denotes a *DV* quantity, such as a dollar loss or down time, in which case it represents the mean total annual dollar loss or the mean total annual down time, respectively. These estimates clearly would be valuable in performance-based design or retrofit decisions. The quantity $\mathrm{E}[\Sigma X]$ may also be useful for certain *DM* quantities, such as the dissipated energy or accumulated plastic strain or rotation in a member or joint. However, for an *EDP* quantity, the above measure may not be of much value. For example, if X denotes an interstory drift, then $\mathrm{E}[\Sigma X]$ is the mean of the sum of all interstory drift values occurring in one year. This measure has little relevance to reliability or safety of the structure.

In summary, we have shown that the function $\lambda(x)$, where x may denote a *DV*, *DM* or *EDP* threshold, offers a wealth of information beyond the mean frequency of $\{x < X\}$ events, which can be useful in performance-based earthquake engineering. Specifically, this function can be used to compute the distribution of X for a random earthquake, as well as an approximation of the distribution of the extreme value of X for all earthquakes occurring during a given period of time. Furthermore, this function can be used to compute the expected cumulative value of X values for all earthquakes in a given period of time. It is noted that, while the lower limit of $\lambda(x)$ may not be important for computing the distribution of the extreme value X_T, it is clearly important for computing the expected cumulative value $\mathrm{E}[\Sigma X]$ in (6). In fact, as Figure 1 suggests, the left end of $\lambda(x)$ dominates the area underneath the curve. The use of λ_0 as a benchmark for determining $\lambda(0)$, as described in the beginning of this section, is useful for accurately determining $\lambda(x)$ for small x values.

Computation of $G(edp \mid im)$

The predominant method for computing the distribution of an *EDP* for a given intensity threshold $IM = im$ is based on nonlinear time history analysis using recorded ground motions, commonly known as Incremental Dynamic Analysis or IDA (Vamvatsikos and Cornell 2002). For the given *im*, a disaggregation of the hazard leads to a pair of predominant magnitude and distance. Recorded ground motions are then selected that have similar magnitude and distance characteristics. The recorded motions are scaled to have the same *im* level. For example, if *im* represents the spectral acceleration at the fundamental period of the structure, then the recorded ground motions are all scaled to produce the same spectral acceleration at that frequency. Nonlinear structural dynamic analysis is then performed for the ensemble of selected recorded ground motions and the *edp* value for each record is determined. The median and coefficient of variation (c.o.v.) of the sample of *edp* values are estimated and a lognormal distribution is fitted to these statistics. Usually the same set of records is used for a range of *im* levels with varying scaling factors. The IDA has proven to be an effective method of approximate probabilistic analysis and has gained considerable popularity among researchers and practitioners interested in PBEE. The discussion in this section deals not with the method of analysis in IDA, but with the choice of recorded ground motions for the analysis.

In a parametric study such as IDA, it would be convenient to use simulated ground motions, or a stochastic representation of the ground motion. The choice for recorded ground motions is made, primarily because the design profession views simulated ground motions with suspicion and as not representing real earthquakes. In the author's opinion, one could equally question whether scaled versions of recorded ground motions are realistic representations of earthquakes. However, there are other problems with the choice of recorded motions, as described below, which make the option of using simulated ground motions, or a stochastic representation of the ground motion, worthy of consideration.

The site of a given structure has its unique characteristics. These include the local soil conditions, the geologic setting of the site, the position of the site relative to seismic sources, and the characteristics of the surrounding ground, which forms the medium through which seismic waves propagate. It would be ideal if one would have a large set of recorded ground motions at the site of interest, from which one could select the desired sample of ground motions for each *im* level for IDA. Such a sample would be inherently consistent with the specific characteristics of the site. However, this is not the case, as no individual building site has a sufficient number of recorded ground motions to provide a workable sample. As a result, when the choice is limited to recorded ground motions, one is forced to use accelerograms that are recorded at many different sites. The problem is that such a sample of ground motions includes the variation inherent in the characteristics of the recording sites, which is irrelevant to the site of interest. In other words, the sample of multiple-site recorded ground motions potentially has more variability than one would expect from the sample ground motions at a given site. The result is a potential overestimation of the variability in the distribution of *EDP* for a given *im*. In contrast, a stochastic model of the ground motion can be constructed that is specific to the site of interest. Either simulation or nonlinear stochastic dynamics can then be used to compute the conditional distribution of the *EDP*.

To test the above hypothesis, the six-story building model in Figure 3 is considered. The building has nonlinear story stiffnesses, which are modeled by the Bouc-Wen hysteresis law. The hysteresis loops for the 1st and 6th story columns are also shown in Figure 3. Following the conventional approach, 10 recorded ground motions are selected (same as those used by Moehle *et al*. 2005) and are scaled to have the same spectral accelerations at the fundamental period of the structure based on its initial stiffness, which is 0.6 s. The scaled pseudo-acceleration spectra of the selected ground motions can be seen in Figure 4 (left), where the average of the 10 spectra is also shown. The selected *im* is the spectral acceleration of 1g at 0.6s period. As the second alternative, the ground motion at the site is modeled as a stochastic process having 10s stationary strong-motion duration with a power spectral density characterizing the site of interest. Three models are considered: (a) a white noise excitation characterizing a rock site with a very broad spectrum of frequencies, (b) a wide-band Kanai-Tajimi power spectrum characterizing a firm ground with a predominant frequency of 2.5 Hz and a bandwidth parameter of 0.6, and (c) a narrow-band Kanai-Tajimi power spectrum characterizing a soft ground with a predominant frequency of 1.5 Hz and a bandwidth parameter of 0.3. The average pseudo-acceleration spectra of these motions, scaled at the fundamental period of the structure, are shown in Figure 4 (right) together with the average spectrum of the recorded motions.

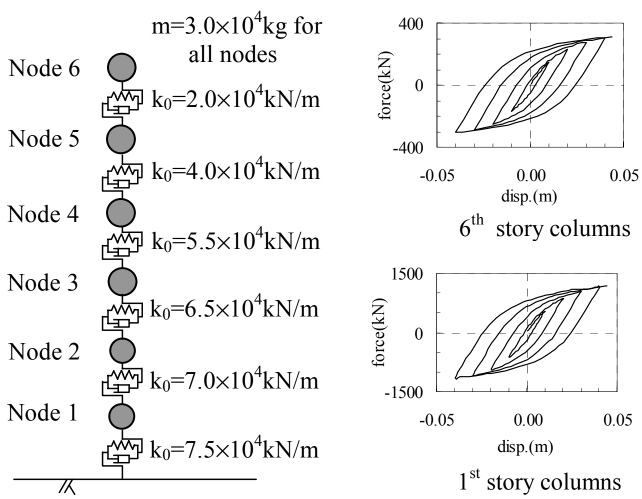

Figure 3 Model of 6-story building with hysteresis loops of 1st and 6th story columns.

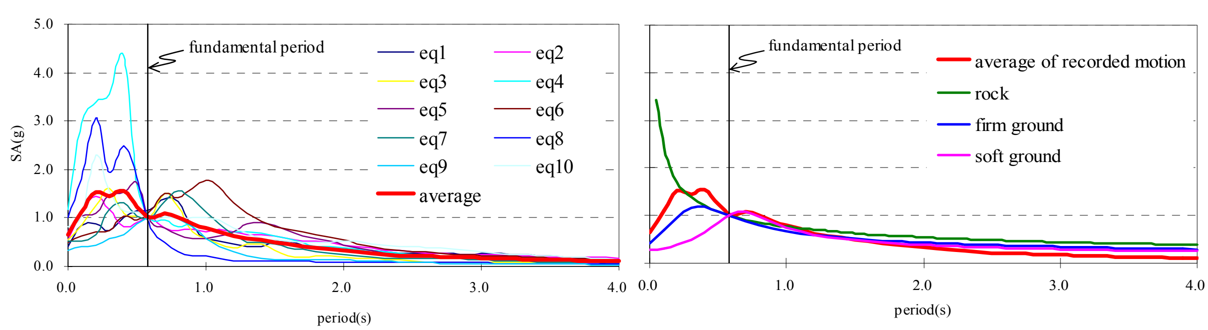

Figure 4 Scaled pseudo-acceleration spectra of recorded (left) and stochastic (right) ground motions.

Table 1 shows the estimated mean and c.o.v. of two selected *EDP*s, i.e., the peak values of the 1st and 6th interstory drifts, based on the two approaches. The statistics based the recorded motions are estimated from the sample of 10 nonlinear dynamic analysis using the selected recorded ground motions. The statistics for each of the stochastic inputs are computed by a nonlinear random vibration analysis method, the details of which will not be described here, except to stress that the accuracy of these estimates have been verified by Monte Carlo simulations.

Table 1. *EDP* statistics based on recorded and stochastic ground motions.

EDP	Input motion		Mean, m	C.o.v.
1st inter-story drift	Recorded motions		0.0263	0.390
	Stochastic:	rock site	0.0252	0.206
		firm site	0.0215	0.203
		soft site	0.0210	0.226
6th inter-story drift	Recorded motions		0.0238	0.334
	Stochastic:	rock site	0.0385	0.164
		firm site	0.0307	0.179
		soft site	0.0199	0.176

Two observations in Table 1 are noteworthy. First, we observe a variation in the mean *EDP* values depending on the stochastic model selected for the site. This is, of course, expected, since the characteristics of the site must surely influence the *EDP* value. For example, the mean value of the 6th interstory drift is much smaller for the soft site, because the motion for this site is deficient in higher frequencies that significantly contribute to this response. This kind of differentiation obviously is not possible with the recorded ground motions, unless one is able to select recorded motions that accurately reflect the conditions at the site of interest. The second important observation is that, regardless of the stochastic model used for the site, the c.o.v. of the *EDP* estimates based on the recorded ground motions is about twice the estimated c.o.v. based on the stochastic models. It is argued here that this large variability in the *EDP* estimate is partly due to the mixing of recorded ground motions from different sites. Such an overestimation of the variability could have a significant influence on the computed metrics for PBEE.

Another shortcoming of using recorded ground motions has to do with the robustness of the results. Specifically, independent analysts are likely to arrive at different estimates of the distribution of the *EDP*, depending on their selected recorded motions. With the stochastic model, once the model is selected, any independent analyst should arrive at the same result.

In summary, the conventional method of using recorded ground motions for the estimation of the conditional distribution of an *EDP* for a given *im* may grossly overestimate the variability in the *EDP* due to the mixing of ground motions from different sites. Furthermore, with this approach, it is not easy to account for the specific characteristics of the site, as recorded ground motions consistent with those characteristics may not be available. As an alternative, it is argued that a stochastic characterization of the ground motion specific for the site of interest may lead to a more realistic estimate of the distribution of *EDP*. This approach will also produce a more robust estimate of the distribution of an *EDP*.

Acknowledgment

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